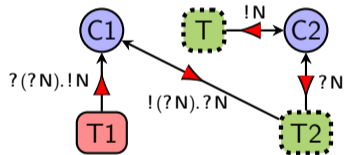
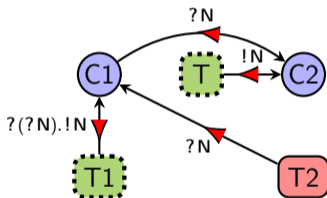


Connectivity Graphs: A Method for Proving Deadlock Freedom Based on Separation Logic

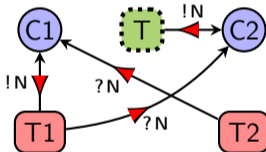
Jules Jacobs¹



Robbert Krebbers¹



Stephanie Balzer²



¹Radboud University Nijmegen

²Carnegie Mellon University

Safety in Iris

Iris safety theorem: **all threads can always step**

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- ▶ Spinlock CAS loops can always step...but **deadlock not ruled out**

We wanted: **deadlock-free Iris**

We got: progress & preservation style proof of deadlock freedom for session types

- ▶ Uses separation logic
- ▶ **Maybe the techniques help toward deadlock free Iris**

Session types

Message passing concurrency with first-class channels (Honda [1993])

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GV: functional programming with session types

(Gay and Vasconcelos [2010], Wadler [2012])

$$\text{fork} : (s \multimap 1) \rightarrow \bar{s}$$
$$\text{send} : (!t. s) \times t \multimap s$$
$$\text{close} : \text{End} \multimap 1$$
$$\text{receive} : ?t. s \multimap s \times t$$
$$\text{let } c = \text{fork}(\lambda c'. \dots \text{receive}(c') \dots) \text{ in } \text{send}(c, 23) \dots$$

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Two owners per channel

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Graph reasoning intertwined with language specifics.

Encapsulating the graph reasoning made it manageable.

Contribution: connectivity graph proof method

This work: **connectivity graphs**

- ▶ Method for factoring out graph reasoning from reasoning about typing
- ▶ Mechanized in the Coq proof assistant
- ▶ Applied to prove deadlock freedom for feature-rich session-typed language
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Run-time configuration ρ

Threads: $\{T_1 \mapsto e_1, \dots, T_6 \mapsto e_6\}$

Channels: $\{C_1 \mapsto \text{buf}_1, \dots, C_5 \mapsto \text{buf}_5\}$

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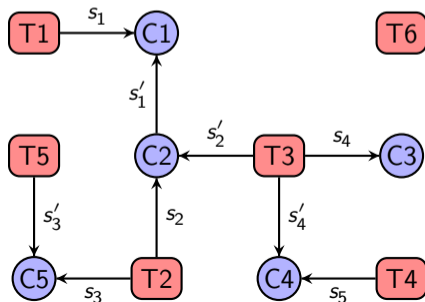
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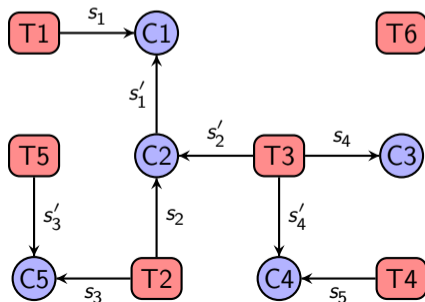
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$$G \vDash \rho$$
$$wf(\rho) := \exists G. G \vDash \rho$$

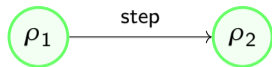
Connectivity graph G



Connectivity graph proof based on progress and preservation

ρ_1

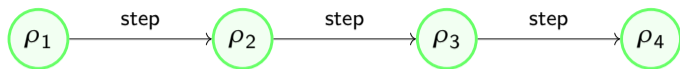
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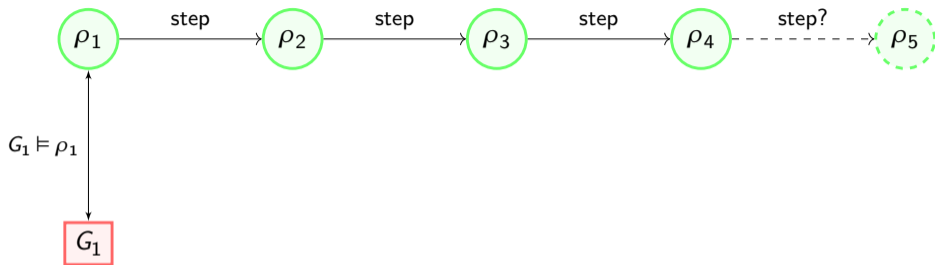
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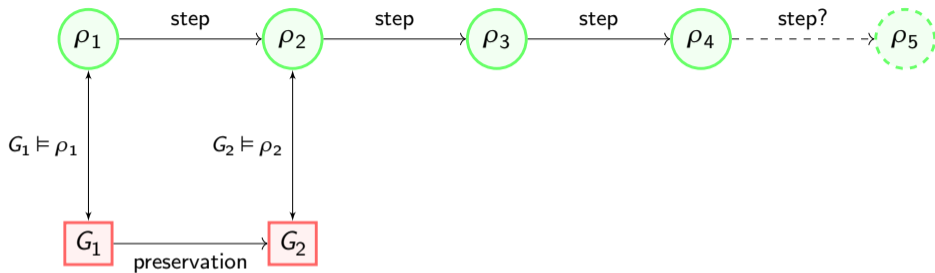
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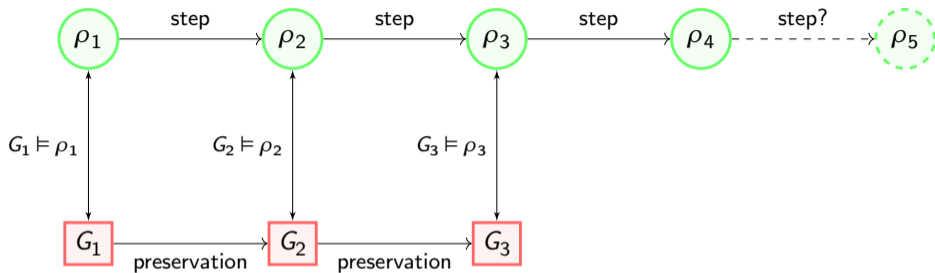
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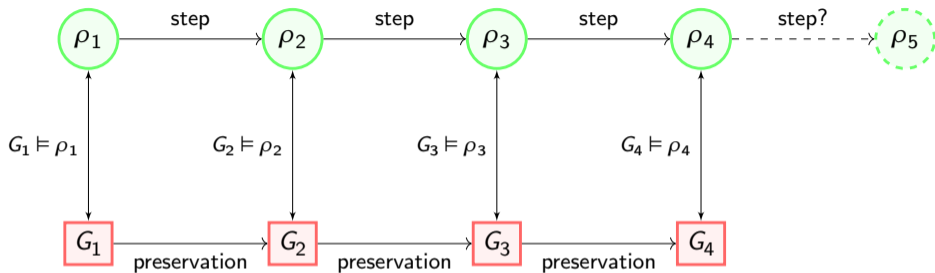
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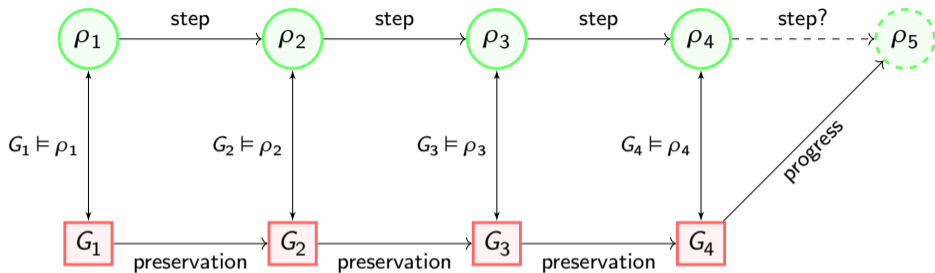
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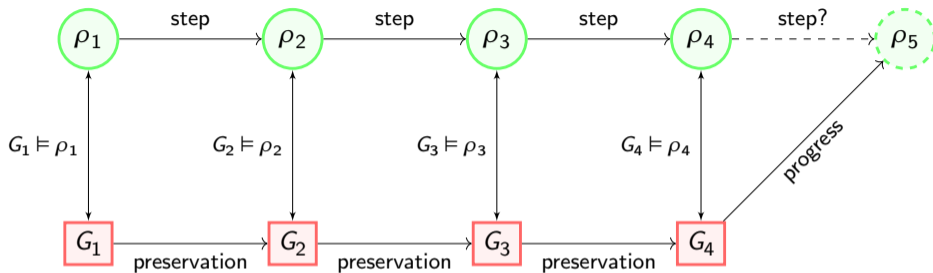
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Connectivity graph framework:

- ▶ $Cgraph(V, L)$ data type for acyclic labeled graphs
- ▶ Generic construction for $wf(\rho) := \overline{wf(P_\rho)}$
 - ▶ Parameterized by local separation logic predicate $P_\rho(v)$ for each vertex $v \in G$
- ▶ Preservation: graph transformations in separation logic
- ▶ Progress: waiting induction principle for $Cgraph(V, L)$

All generic over vertices V and labels L

Linear heap typing in separation logic: (cf. Rouvoet [2020]'s definitional interpreters)

$$\frac{\Sigma_1 \vdash e_1 : \tau_1 \quad \Sigma_2 \vdash e_2 : \tau_2 \quad \Sigma_1 \cap \Sigma_2 = \emptyset}{\Sigma_1 \cup \Sigma_2 \vdash (e_1, e_2) : \tau_1 \times \tau_2}$$

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Lemmas in separation logic:

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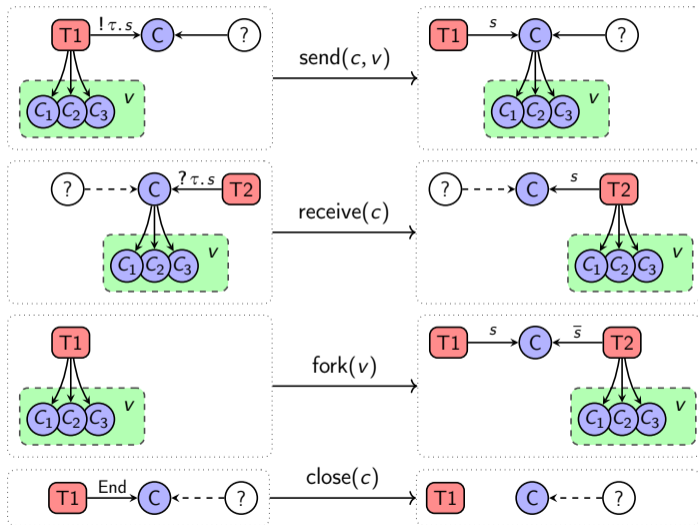
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 &\qquad \qquad \qquad \Rightarrow \\
 (K[e] : B) &\dashv\vdash \exists A. (e : A) * \forall e'. (e' : A) \multimap (K[e'] : B)
 \end{aligned}$$

We use the Iris proof mode to reason in separation logic (Krebbers et al. [2017])

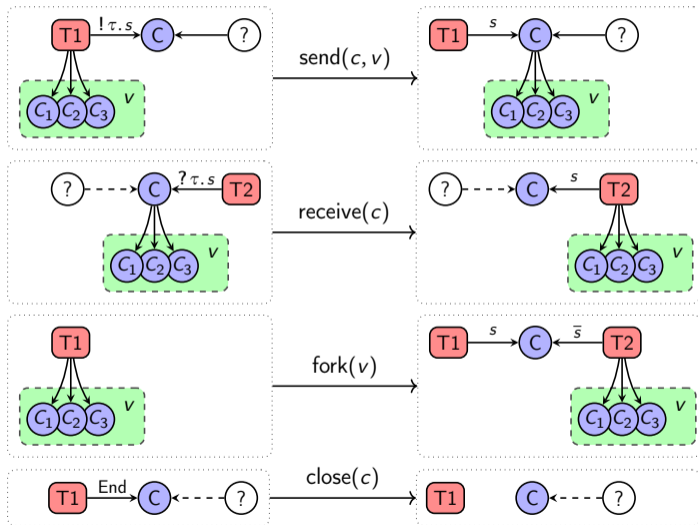
Preservation via local graph transformations



Preserves:

- ▶ Acyclicity
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In separation logic: if

$$P_\rho(T_1) * (\text{own}(C \mapsto s) \multimap P_\rho(C)) \vdash$$

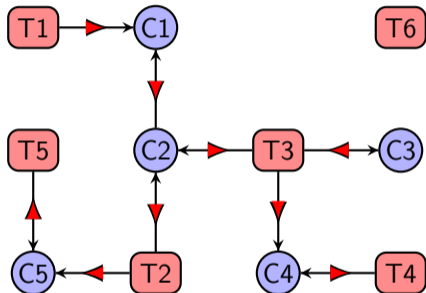
$$(\text{own}(C \mapsto s') \multimap P_{\rho'}(T_1)) * P_{\rho'}(C)$$

then: $\overline{wf}(P_\rho) \rightarrow \overline{wf}(P_{\rho'})$

Explained in our paper!

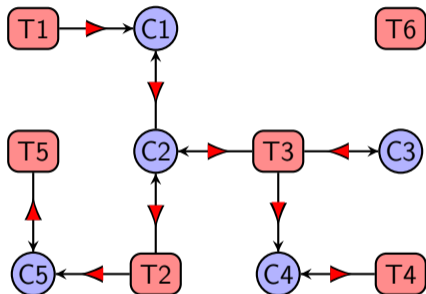
Progress via waiting induction

Connectivity graph with *waiting dependencies* (▶)
derived from run-time configuration



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Lemma (Waiting induction)

Let $R(v, w)$ be any relation on the vertices. To prove $P(v)$, we may assume $P(w)$ for all w such that $v \rightarrow w$ and $R(v, w)$, or $w \rightarrow v$ and $\neg R(w, v)$

Mechanization

Mechanization in Coq:

- ▶ Generic *Cgraph*(V, L) library: 4999 LOC
- ▶ Channels + unrestricted & recursive types language definition: 451 LOC
- ▶ Language specific deadlock and leak freedom proof: 1688 LOC

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MPGV: linear λ -calculus with multiparty session types

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<https://github.com/julesjacobs/cgraphs>

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Ideas? Questions?

julesjacobs@gmail.com

Extra slides

Stronger deadlock and leak freedom result

Global progress is the standard notion that people use

Our POPL reviewers: Can your method prove something stronger?

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Theorem. For well-typed initial programs, no partial deadlock occurs

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