Later Credits

Resourceful Reasoning for the Later Modality

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The Historic Foundation of Iris

Iris

Step-Indexed Logical Relations

Separation Logic

A Powerful Combination

Example: RustBelt

- **step-indexing** for **recursive** types
- **separation logic** for **ownership** types

Step-Indexing: A Double Edged Sword

Step-indexing enables **recursive reasoning**

Löb induction, higher-order ghost state, ...

but introduces irritating **step-indexing artifacts**.

the later modality $\triangleright P$

Running Example: Impredicative Invariants

Opening Invariants (from Iris 1.0)

$$\frac{\{P * R\}\, e\, \{Q * R\} \qquad e \text{ atomic}}{\boxed{R} \vdash \{P\}\, e\, \{Q\}}$$

Running Example: Impredicative Invariants

Actually …

later modality masks

$$\frac{\{P * \triangleright R\}\ e\ \{v.\ Q * \triangleright R\}_{\mathcal{E}\setminus\mathcal{N}} \qquad e \text{ atomic} \qquad \mathcal{N} \subseteq \mathcal{E}}{\boxed{R}^{\mathcal{N}} \vdash \{P\}\ e\ \{v.\ Q\}_{\mathcal{E}}}$$

because invariants in Iris **are step-indexed**.

The Akward Role of the Later Modality

The later modality prevents **inconsistent proofs** ($\triangleright R$ is sound, R not necessarily),

but in proofs **we worry mostly about removing it** (we want R, not $\triangleright R$).

Example: A Typical Iris Proof

$$\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n} \vdash \{\mathsf{True}\}\ !\ell\ \{v.\, v \in \mathbb{N}\}$$

Example: A Typical Iris Proof

$$\frac{\vdash \{\triangleright(\exists n : \mathbb{N}.\, \ell \mapsto n)\}\ !\ell\ \{v.\, v \in \mathbb{N} * \triangleright(\exists n : \mathbb{N}.\, \ell \mapsto n)\}}{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n} \vdash \{\mathsf{True}\}\ !\ell\ \{v.\, v \in \mathbb{N}\}}$$

Example: A Typical Iris Proof

no more later

$$\dfrac{\dfrac{\vdots}{\vdash \{(\exists n:\mathbb{N}.\, \ell \mapsto n)\}\, !\ell\, \{v.\, v \in \mathbb{N} * \triangleright(\exists n:\mathbb{N}.\, \ell \mapsto n)\}}}{\dfrac{\vdash \{\triangleright(\exists n:\mathbb{N}.\, \ell \mapsto n)\}\, !\ell\, \{v.\, v \in \mathbb{N} * \triangleright(\exists n:\mathbb{N}.\, \ell \mapsto n)\}}{\boxed{\exists n:\mathbb{N}.\, \ell \mapsto n} \vdash \{\mathsf{True}\}\, !\ell\, \{v.\, v \in \mathbb{N}\}}}$$

We have to solve …

The Later Elimination Problem

We have $\triangleright R$ in our context, but we need R to proceed.

Existing Options

- Timeless Propositions
- Commuting Rules
- Program Steps

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We have $\triangleright R$ in our context, but we need R to proceed.

Existing Options

- Timeless Propositions

$$\frac{\{P * R\}\ e\ \{v.\ Q\} \qquad \text{timeless}(R)}{\{P * \triangleright R\}\ e\ \{v.\ Q\}} \qquad\qquad \text{timeless}(\ell \mapsto v)$$

- Commuting Rules
- Program Steps

We have to solve ...

The Later Elimination Problem

We have $\triangleright R$ in our context, but we need R to proceed.

Existing Options

- Timeless Propositions
- Commuting Rules

$$\triangleright(P * Q) \vdash \triangleright P * \triangleright Q \qquad \triangleright(\exists x.\, P) \vdash \exists x.\, \triangleright P \qquad \ldots$$

- Program Steps

We have to solve ...

The Later Elimination Problem

We have $\triangleright R$ in our context, but we need R to proceed.

Existing Options

- Timeless Propositions
- Commuting Rules
- Program Steps

$$\frac{\{R\}\ e'\ \{v.\ Q\} \qquad e \rightarrow_{\text{pure}} e'}{\{\triangleright R\}\ e\ \{v.\ Q\}} \qquad \ldots$$

Limitations of the Existing Options

Existing options apply to most invariants

$$\boxed{R} = \boxed{\exists n : \mathbb{N}.\, \ell \mapsto n} \quad \text{where} \quad \underbrace{\exists n : \mathbb{N}.\, \ell \mapsto n}_{\text{timeless}}$$

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But they are no silver bullet. They do not apply to

$$\boxed{R} = \boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}} \quad \text{where} \quad \underbrace{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}}_{\text{not timeless}}$$

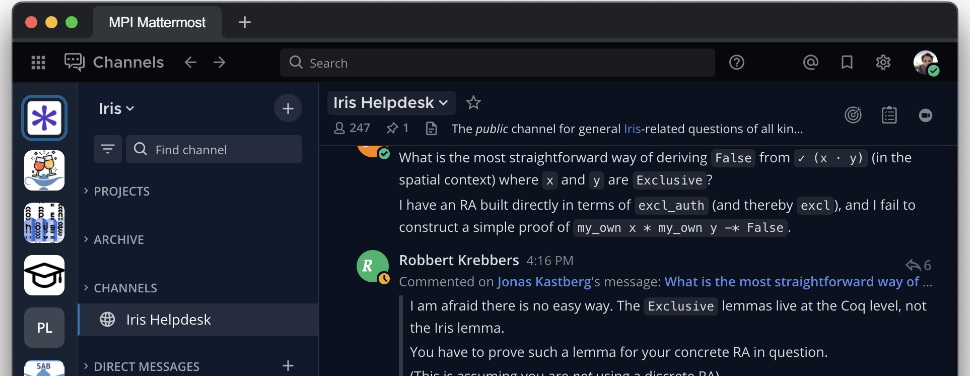
We are stuck ...

invariant guarded by a later

$$\frac{\vdash \{\triangleright \boxed{(\exists n : \mathbb{N}.\, \ell \mapsto n)}\}\ !\ell\ \{v.\, v \in \mathbb{N} * \triangleright \boxed{(\exists n : \mathbb{N}.\, \ell \mapsto n)}\}}{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}} \vdash \{\mathsf{True}\}\ !\ell\ \{v.\, v \in \mathbb{N}\}}$$

So what then?

> “Help …



So what then?

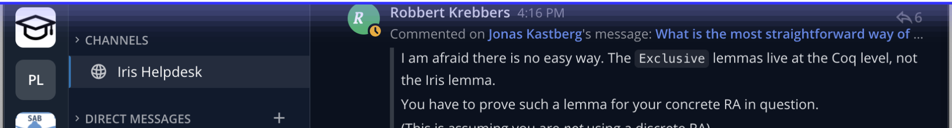
> "Help ...

Have you tried these **non-local refactorings** of your proof

- flattening your invariant hierarchy

⋮

or **considered giving up?**



Developing a Fourth Option

Step-Indexed
Logical
Re

Separation
Logic

How about using this pillar to develop another option?

Our Contribution: Later Credits

Later credits turn

the right to eliminate a later into an

transform $\triangleright R$ into R

ownable resource, which is subject to

a later credit $£1$

traditional separation logic reasoning.

passing around, framing, sharing via invariants

Later Credits in a Nutshell

$$\frac{\{R\}\, e'\, \{v.\, Q\} \qquad e \rightarrow_{\mathsf{pure}} e'}{\{\triangleright R\}\, e\, \{v.\, Q\}}$$

becomes

$$\frac{\{R * \pounds\, 1\}\, e'\, \{v.\, Q\} \qquad e \rightarrow_{\mathsf{pure}} e'}{\{R\}\, e\, \{v.\, Q\}} \qquad \frac{\{R\}\, e\, \{v.\, Q\}}{\{\pounds\, 1 * \triangleright R\}\, e\, \{v.\, Q\}}$$

Novelty: Prepaid Reasoning

$$\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}}\}\ f(\underline{41 + 1});\ \underline{!\ell}\ \{v.\, v \in \mathbb{N}\}$$

we obtain $\pounds\,1$

we spend $\pounds\,1$

Prepaid Reasoning in Action

$$\frac{}{\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}}\}\ f(41+1);\, !\ell\ \{v.\, v \in \mathbb{N}\}}$$

Prepaid Reasoning in Action

$$\frac{\left\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}} * \pounds\,1\right\}\, f(42);\, !\ell\, \{v.\, v \in \mathbb{N}\}}{\left\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}}\right\}\, f(41+1);\, !\ell\, \{v.\, v \in \mathbb{N}\}}$$

Prepaid Reasoning in Action

$$\frac{\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}} * \pounds 1\}\ f(42); !\ell\ \{v.\, v \in \mathbb{N}\}}{\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}}\}\ f(41+1); !\ell\ \{v.\, v \in \mathbb{N}\}}$$

Prepaid Reasoning in Action

$$\dfrac{\dfrac{}{\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}} * \pounds 1\}\ !\ell\ \{v.\, v \in \mathbb{N}\}}}{\dfrac{\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}} * \pounds 1\}\ f(42); !\ell\ \{v.\, v \in \mathbb{N}\}}{\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}}\}\ f(41 + 1); !\ell\ \{v.\, v \in \mathbb{N}\}}}$$

Prepaid Reasoning in Action

$$\dfrac{\dfrac{\dfrac{\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}} * \pounds 1\}\; f(41+1); !\ell\; \{v.\, v \in \mathbb{N}\}}{\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}} * \pounds 1\}\; f(42); !\ell\; \{v.\, v \in \mathbb{N}\}}}{\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}} * \pounds 1\}\; !\ell\; \{v.\, v \in \mathbb{N}\}}}{\{\triangleright \boxed{(\exists n : \mathbb{N}.\, \ell \mapsto n)} * \pounds 1\}\; !\ell\; \{v.\, v \in \mathbb{N} * \triangleright \boxed{(\exists n : \mathbb{N}.\, \ell \mapsto n)}\}}$$

Prepaid Reasoning in Action

we spend our credit

$$\frac{\dfrac{\dfrac{\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}}\}\; f(41+1); !\ell\; \{v.\, v \in \mathbb{N}\}}{\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}} * £\,1\}\; f(42); !\ell\; \{v.\, v \in \mathbb{N}\}}}{\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}} * £\,1\}\; !\ell\; \{v.\, v \in \mathbb{N}\}}}{\{\triangleright \boxed{(\exists n : \mathbb{N}.\, \ell \mapsto n)} * £\,1\}\; !\ell\; \{v.\, v \in \mathbb{N} * \triangleright \boxed{(\exists n : \mathbb{N}.\, \ell \mapsto n)}\}}$$

Prepaid Reasoning in Action

$$\{\ \boxed{(\exists n : \mathbb{N}.\, \ell \mapsto n)}\ \}\ !\ell\ \{v.\, v \in \mathbb{N} * \triangleright \boxed{(\exists n : \mathbb{N}.\, \ell \mapsto n)}\ \}$$

$$\{\triangleright \boxed{(\exists n : \mathbb{N}.\, \ell \mapsto n)} * £\,1\}\ !\ell\ \{v.\, v \in \mathbb{N} * \triangleright \boxed{(\exists n : \mathbb{N}.\, \ell \mapsto n)}\ \}$$

$$\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}} * £\,1\}\ !\ell\ \{v.\, v \in \mathbb{N}\}$$

$$\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}} * £\,1\}\ f(42);\, !\ell\ \{v.\, v \in \mathbb{N}\}$$

$$\{\boxed{\boxed{\exists n : \mathbb{N}.\, \ell \mapsto n}}\}\ f(41 + 1);\, !\ell\ \{v.\, v \in \mathbb{N}\}$$

Application: Prepaid Invariants

sharing later credits via invariants

Application: Logical Atomicity

cleaning up existing proofs

Theory and Soundness

the intuition on a napkin

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Do we really need a later?

no later

$$\frac{\{P * R\}\, e\, \{v.\, Q * R\} \qquad e \text{ atomic}}{\boxed{R} \vdash \{P\}\, e\, \{v.\, Q\}}$$

“ That cannot be sound, can it?

Later Credits in Invariants

Idea: We **prepay** the later elimination

$$\boxed{R}_{\mathsf{pre}} \triangleq \boxed{R * £\,1}$$

such that we get **direct access** to R.

$$\frac{\{R * P\}\; e\; \{v.\, Q * R * £\,1\} \qquad e \text{ atomic}}{\boxed{R}_{\mathsf{pre}} \vdash \{P\}\; e\; \{v.\, Q\}}$$

Later Credits in Invariants

Idea: We **prepay** the later elimination

$$\boxed{R}_{\text{pre}} \triangleq \boxed{R * £\,1}$$

such that we get **direct access** to R.

generated by the next step

$$\frac{\{R * P\}\, e\, \{v.\, Q * R * £\,1\} \qquad e \text{ atomic}}{\boxed{R}_{\text{pre}} \vdash \{P\}\, e\, \{v.\, Q\}}$$

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$$\dfrac{\dfrac{\dfrac{\dfrac{\{R * P\}\, e\, \{v.\, Q * R * £\,1\}}{\{£\,1 * \triangleright(R * P)\}\, e\, \{v.\, Q * R * £\,1\}}\text{ spend credit}}{\{\triangleright £\,1 * \triangleright(R * P)\}\, e\, \{v.\, Q * R * £\,1\}}\text{ timelessness of } £\,n}{\{P * \triangleright(R * £\,1)\}\, e\, \{v.\, Q * \triangleright(R * £\,1)\}}\text{ later shuffling}}{\boxed{R}_{\mathsf{pre}} \vdash \{P\}\, e\, \{v.\, Q\}}\text{ open invariant}$$

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$$\dfrac{\dfrac{\dfrac{\dfrac{\{R * P\}\; e\; \{v.\, Q * R * £\,1\}}{\{£\,1 * \triangleright(R * P)\}\; e\; \{v.\, Q * R * £\,1\}}\ \text{spend credit}}{\{\triangleright £\,1 * \triangleright(R * P)\}\; e\; \{v.\, Q * R * £\,1\}}\ \text{timelessness of } £\,n}{\{P * \triangleright(R * £\,1)\}\; e\; \{v.\, Q * \triangleright(R * £\,1)\}}\ \text{later shuffling}}{\boxed{R}_{\text{pre}} \vdash \{P\}\; e\; \{v.\, Q\}}\ \text{open invariant}$$

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Later Credits in Invariants

Idea: We **prepay** the later elimination

$$\boxed{R}^{\text{pre}} \triangleq \boxed{R * \pounds 1}$$

such that we get **direct access** to R.

$$\cfrac{\cfrac{\cfrac{\cfrac{\{R * P\}\, e\, \{v.\, Q * R * \pounds 1\}}{\{\pounds 1 * \triangleright(R * P)\}\, e\, \{v.\, Q * R * \pounds 1\}}\ \text{spend credit}}{\{\triangleright \pounds 1 * \triangleright(R * P)\}\, e\, \{v.\, Q * R * \pounds 1\}}\ \text{timelessness of } \pounds n}{\{P * \triangleright(R * \pounds 1)\}\, e\, \{v.\, Q * \triangleright(R * \pounds 1)\}}\ \text{later shuffling}}{\boxed{R}^{\text{pre}} \vdash \{P\}\, e\, \{v.\, Q\}}\ \text{open invariant}$$

Prepaid Invariants

In fact, we obtain

no later

$$\frac{\{P * R\}\, e\, \{v.\, Q * R\} \qquad e \text{ atomic}}{\boxed{R}_{\text{pre}} \vdash \{P\}\, e\, \{v.\, Q\}}$$

Disclaimer 1. To obtain this rule, we need to generate more than one credit per step. To do so, we modify Jourdan's multiple-laters-per-step extension of Iris.

Disclaimer 2. The paradox is of course still true. Even with later credits, we cannot open invariants without a guarding later around updates.

Application: Prepaid Invariants

sharing later credits via invariants

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cleaning up existing proofs

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Logical Atomicity ...

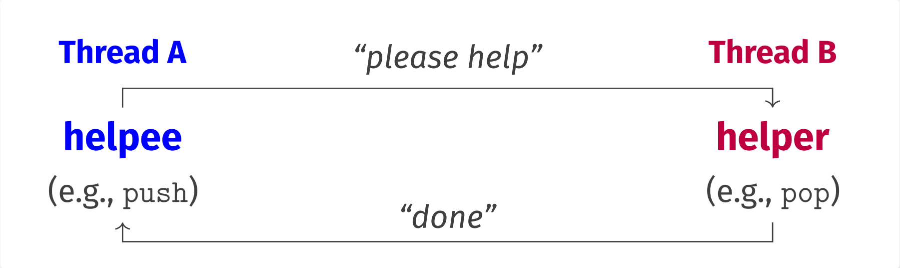
... in a nutshell:

relaxed to "logically atomic" instructions

$$\frac{\{P * R\}\, e\, \{v.\, Q * R\} \qquad e \text{ atomic}}{\boxed{R} \vdash \{P\}\, e\, \{v.\, Q\}}$$

The later troubles …

… arise for **data structures with helping.**



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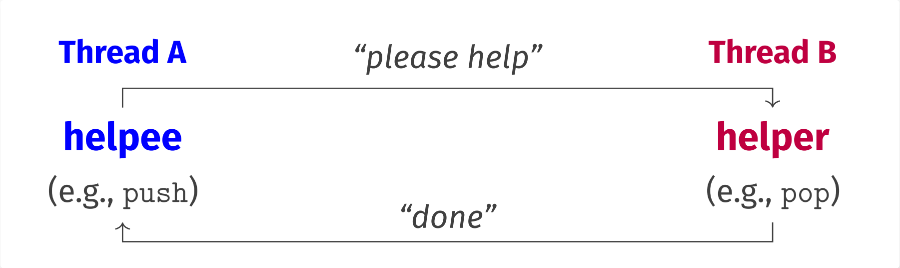
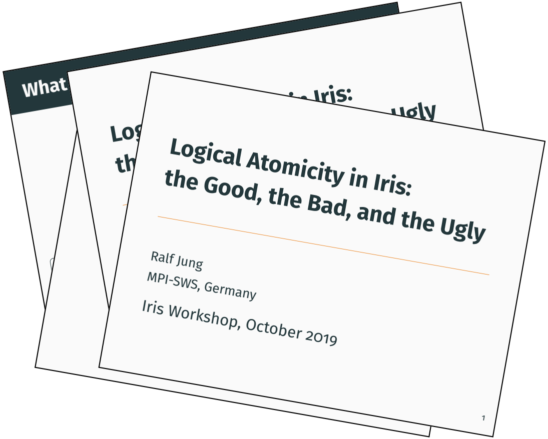


Complication. The interaction physically happens through memory, and logically happens **through invariants**.

How does it work?

Ask Ralf!

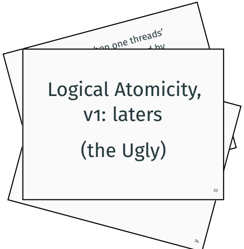


The Main Takeaway

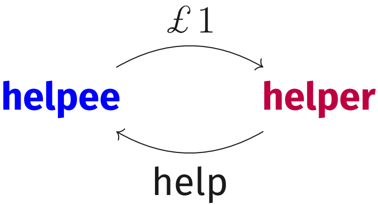
Later credits remove the **ugly parts of** logical atomicity.

`laterable`

without later credits



with later credits



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The Later Credit Mechanism

A resource $\pounds\, n$

$$\pounds\,(n+m) \dashv\vdash \pounds\, n * \pounds\, m \qquad \text{timeless}(\pounds\, n)$$

an update $\Rrightarrow_{\mathsf{le}} P$

$$P \vdash \Rrightarrow_{\mathsf{le}} P \qquad \Rrightarrow_{\mathsf{le}} P * (P -\!\!* \Rrightarrow_{\mathsf{le}} Q) \vdash \Rrightarrow_{\mathsf{le}} Q \qquad \pounds\, 1 * \triangleright P \vdash \Rrightarrow_{\mathsf{le}} P$$

a monad

and Hoare rules

$$\frac{\{P\}\, e\, \{v.\, Q\}}{\{\Rrightarrow_{\mathsf{le}} P\}\, e\, \{v.\, Q\}} \qquad \frac{\{P * \pounds\, 1\}\, e'\, \{v.\, Q\} \qquad e \to_{\mathsf{pure}} e'}{\{P\}\, e\, \{v.\, Q\}}$$

Soundness

Observation. Adequacy in Iris is only concerned with the **amortized number** of later eliminations.

without credits $e_0 \xrightarrow{\triangleright\ \text{elim.}} e_1 \xrightarrow{\triangleright\ \text{elim.}} \cdots \xrightarrow{\triangleright\ \text{elim.}} e_n$

at most n later eliminations

with credits $e_0 \xrightarrow{\pounds\,1} e_1 \xrightarrow{\pounds\,1} \cdots \xrightarrow{\pounds\,1} e_n$

Our Contribution: Later Credits

Later credits turn

the right to eliminate a later into an

transform $\triangleright R$ into R

ownable resource, which is subject to

a later credit $\pounds 1$

traditional separation logic reasoning.

passing around, framing, sharing via invariants

Using Later Credits

Step 1. Replace $\Rrightarrow P$ with $\Rrightarrow_{\mathsf{le}} P$ in your definitions.[1]

Step 2. Profit

- ✓ in program verification proofs
- ✓ in logical relation constructions
- ✓ in ghost theories
- ✓ in logical atomicity proofs

[1] Mostly backwards compatible. Missing interaction rules with plain propositions.

Later Credits vs. Time Receipts

Time receipts track **the number of laters per step.**

$$e_0 \xrightarrow{\quad\triangleright\quad} e_1 \xrightarrow{\quad\triangleright^2\quad} \cdots \xrightarrow{\quad\triangleright^n\quad} e_n$$

Later credits control **where laters are.**

$$£\,1 * \triangleright P \vdash \mathrel{\models\!\!\!\Rrightarrow}_{\mathsf{le}} P \qquad \text{and} \qquad \frac{\{R\}\; e\; \{v.\, Q\}}{\{£\,1 * \triangleright R\}\; e\; \{v.\, Q\}}$$

Later Credits + Time Receipts

We add time receipts $⧗ n$

$$\frac{\{P * £\,1 * ⧗ 1\}\; e_2\; \{v.\, Q\} \qquad e_1 \rightarrow_{\text{pure}} e_2}{\{P\}\; e_1\; \{v.\, Q\}} \qquad \frac{\{P\}\; e\; \{v.\, Q\} \qquad e \notin \textit{Val}}{\{P * ⧗ n\}\; e\; \{v.\, Q * £\,n * ⧗ n\}}$$

by integrating with **Jourdan's multiple-laters-per-step extension**. The definition of prepaid invariants becomes $\boxed{R}_{\text{pre}} \triangleq \boxed{R * £\,1 * ⧗ 1}$, satisfying

$$\frac{\boxed{R}_{\text{pre}} \vdash \{P\}\; e\; \{v.\, Q\}}{\{\triangleright R * £\,1 * ⧗ 1 * P\}\; e\; \{v.\, Q\}} \qquad \frac{\{P * R\}\; e\; \{v.\, Q * R\} \qquad e \text{ atomic}}{\boxed{R}_{\text{pre}} \vdash \{P\}\; e\; \{v.\, Q\}}$$

The Later Elimination Update

choose a path

add a later to your goal

$$|\!\!\Rrightarrow_{\mathsf{le}} P \triangleq \forall n.\ \pounds_\bullet n \mathrel{-\!\!*} |\!\!\Rrightarrow((\pounds_\bullet n * P) \vee (\exists m < n.\ \pounds_\bullet m * \triangleright |\!\!\Rrightarrow_{\mathsf{le}} P))$$

ghost state update

credit decrease

where $\pounds\, n \triangleq \boxed{\circ n}^{\gamma_{\mathsf{lc}}}$ and $\pounds_\bullet n \triangleq \boxed{\bullet n}^{\gamma_{\mathsf{lc}}}$ from *Auth*$(\mathbb{N}, +)$.