Outcome Separation Logic
Local Reasoning for Correctness and Incorrectness with Computational Effects

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Since Hoare Logic was introduced, new programming paradigms have arisen:

- **Computational effects**: nondeterminism, randomization, quantum computation, concurrency, …
- **Incorrectness Reasoning**
Overview

Since Hoare Logic was introduced, new programming paradigms have arisen:

- **Computational effects**: nondeterminism, randomization, quantum computation, concurrency, …
- **Incorrectness Reasoning**

- Can a single logical foundation capture it all?
Since Hoare Logic was introduced, new programming paradigms have arisen:

- **Computational effects**: nondeterminism, randomization, quantum computation, concurrency, ...
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Can a single logical foundation capture it all?

What does a Separation Logic based on this look like?
Computational Effects and Incorrectness
Nondeterministic Bugs

- Malloc is nondeterministic, it might return null.

\[
\{\text{ok : emp}\}
\]

\[
x := \text{malloc}() ;
\]

\[
[x] \leftarrow 1
\]

\[
\{(\text{ok : } x \mapsto 1) \lor (\text{er : } x = \text{null})\}
\]

- Does this specification characterize the bug?
Nondeterministic Bugs

- Malloc is nondeterministic, it might return null.

\[
\begin{align*}
\{ \text{ok : emp} \} \\
\quad x := \text{malloc()} ; \\
\quad [x] \leftarrow 1 \\
\quad \{(\text{ok : } x \mapsto 1) \lor (\text{er : } x = \text{null})\}
\end{align*}
\]

- Does this specification characterize the bug?

- **No!** We don’t know if the crash state is reachable
Incorrectness Logic

- Incorrectness Logic Semantics

\[ \models [P] C [Q] \quad \text{iff} \quad \forall \tau \models Q. \quad \exists \sigma \models P. \quad \tau \in [C](\sigma) \]

- Every state satisfying the post is \textit{reachable} from some state satisfying the pre
Incorrectness Logic

- Incorrectness Logic Semantics

\[ \models [P] C [Q] \iff \forall \tau \in Q. \ \exists \sigma \in P. \ \tau \in [C] (\sigma) \]

- Every state satisfying the post is \textit{reachable} from some state satisfying the pre

- Does this spec characterize the bug?

\[
\begin{align*}
&\text{[ok : emp]} \\
&x := \text{malloc()} \diamond \\
&[x] \leftarrow 1 \\
&[(\text{ok : } x \mapsto 1) \lor (\text{er : } x = \text{null})]
\end{align*}
\]
Dropping Disjuncts

- The following (more concise) spec is also valid

\[
\begin{align*}
\text{[ok : emp]} & \quad \text{[ok : emp]} \\
\quad x := \text{malloc()} \uparrow & \quad \quad x := \text{malloc()} \uparrow \\
\quad [x] \leftarrow 1 & \quad \quad [x] \leftarrow 1 \\
\quad (\text{ok : } x \mapsto 1) \lor (\text{er : } x = \text{null}) & \quad \quad [\text{er : } x = \text{null}]
\end{align*}
\]

- Static analyzers do not have to traverse all program paths
Program Logics—Comparison

- Incorrectness Logic:
  - Specify *true* bugs
  - Drop program paths for efficiency
  - …but specialized to nondeterminism

- Hoare Logic:
  - Can be used for correctness
  - Supports many types of effects (e.g., probabilistic programs)
  - …but cannot identify bugs
Incorrectness Logic:
- Specify *true* bugs
- Drop program paths for efficiency
- …but specialized to nondeterminism

Hoare Logic:
- Can be used for correctness
- Supports many types of effects (e.g., probabilistic programs)
- …but cannot identify bugs

Can we get the best of both worlds?
Outcome Logic

- Similar to Hoare Logic, but pre/post refer to collections of states

\[ \models \langle \varphi \rangle C \langle \psi \rangle \quad \text{iff} \quad \forall m. \ m \models \varphi \implies [C] m \models \psi \]

- What is \( m \)?
  - Nontermination: \( m \in \text{State} + \{ \bot \} \)
  - Nondeterminism: \( m \in \text{Set(State)} \)
  - Randomization: \( m \in \text{Dist(State)} \)
  - Exceptions: \( m \in \text{State} + E \)
  - \( \vdots \)
Outcome Assertions

- Syntax

\[ \varphi ::= T \mid T^\oplus \mid \varphi \lor \psi \mid \varphi \oplus \psi \mid \epsilon : P \quad \epsilon ::= \text{ok} \mid \text{er} \]

- Semantics (Nondeterministic Interpretation):

\[ S \models T^\oplus \quad \text{iff} \quad S = \emptyset \]
\[ S \models \varphi \oplus \psi \quad \text{iff} \quad \exists S_1, S_2. \quad S = S_1 \cup S_2 \quad \text{and} \quad S_1 \models \varphi \quad \text{and} \quad S_2 \models \psi \]
\[ S \models (\epsilon : P) \quad \text{iff} \quad S \neq \emptyset \quad \text{and} \quad \forall \sigma \in S. \quad \sigma \models (\epsilon : P) \]
\[ : \]
Specifying a Bug

▷ Does this spec characterize the bug?

\[
\begin{align*}
\langle & \text{ok : emp} \rangle \\
& x := \text{malloc()}; \\
& [x] \leftarrow 1 \\
& \langle (\text{ok : } x \mapsto 1) \oplus (\text{er : } x = \text{null}) \rangle
\end{align*}
\]
Specifying a Bug

Does this spec characterize the bug?

\[
\begin{align*}
\langle \text{ok : emp} \rangle \\
&x := \text{malloc}() \triangleright \\
&[x] \leftarrow 1 \\
&\langle (\text{ok : } x \mapsto 1) \oplus (\text{er : } x = \text{null}) \rangle
\end{align*}
\]

Yes!
Dropping Outcomes

- We can also drop some of the program paths

\[
\langle \text{ok : emp} \rangle \quad \langle (\text{ok : } x \mapsto 1) \oplus (\text{er : } x = \text{null}) \rangle
\]
\[
\begin{align*}
x &:= \text{malloc()} \; ; \\
[x] &\leftarrow 1
\end{align*}
\]

\[
\implies
\langle \text{ok : emp} \rangle \quad \langle (\text{er : } x = \text{null}) \oplus \text{T} \rangle
\]
\[
\begin{align*}
x &:= \text{malloc()} \; ; \\
[x] &\leftarrow 1
\end{align*}
\]
Probabilistic Outcomes

- Outcomes are more general than nondeterminism
- We can reason probabilistically about unreliable network connections

\[
\langle \text{ok : true} \rangle \\
x := \text{ping}(192.0.2.1) \\
\langle (\text{ok : } x = 200) \oplus_{99\%} (\text{er : } x = 500) \rangle
\]
The Frame Rule
The Outcome Separating Conjunction

A new operator $\varphi \otimes F$ is defined inductively:

\[
\begin{align*}
\top \otimes F & \triangleq \top \\
\top^{+} \otimes F & \triangleq \top^{+} \\
(\varphi \lor \psi) \otimes F & \triangleq (\varphi \otimes F) \lor (\psi \otimes F) \\
(\varphi \oplus \psi) \otimes F & \triangleq (\varphi \otimes F) \oplus (\psi \otimes F) \\
(\epsilon : P) \otimes F & \triangleq \epsilon : P \ast F
\end{align*}
\]
The Outcome Logic Frame Rule

- The usual Frame Rule:

\[\begin{array}{c}
\{P\} \quad C \quad \{Q\} \\
\text{mod}(C) \cap \text{fv}(F) = \emptyset
\end{array}\]

\[\quad \Rightarrow \quad \{P \star F\} \quad C \quad \{Q \star F\}\]

- What we want:

\[\begin{array}{c}
\langle \varphi \rangle \quad C \quad \langle \psi \rangle \\
\text{mod}(C) \cap \text{fv}(F) = \emptyset
\end{array}\]

\[\quad \Rightarrow \quad \langle \varphi \otimes F \rangle \quad C \quad \langle \psi \otimes F \rangle\]
Suppose allocation is *deterministic*:

\[
\{\text{emp}\} \ x := \text{alloc}() \ {x = 1}
\]

The *address* of \( x \) is 1, since that is the first address.
Memory Allocation and the Frame Rule

- Suppose allocation is *deterministic*:

  \[
  \{\text{emp}\} \ x := \text{alloc()} \ \{x = 1\}
  \]

- The *address* of \( x \) is 1, since that is the first address

- Now, we use the frame rule:

  \[
  \{\text{emp}\} \ x := \text{alloc()} \ \{x = 1\} \quad \text{FRAME}\n  \]

  \[
  \{y \mapsto 2\} \ x := \text{alloc()} \ \{y \mapsto 2 \land x = 1\}
  \]

- Was this inference valid?
Nondeterminism and the Frame Rule

- Idea: make memory allocation nondeterministic
- We cannot say that $x = 1$, we can only say:

\[
\{\text{emp}\} \ x := \text{alloc()} \ \{ x = 1 \lor x = 2 \lor \ldots \}
\]

- After applying the frame rule, this is still true

\[
\{ y \mapsto 2 \} \ x := \text{alloc()} \ \{ y \mapsto 2 \land (x = 1 \lor x = 2 \lor \ldots) \}
\]

- Problem: OL supports execution models other than nondeterminism
Nondeterminism and the Frame Rule

- Idea: make memory allocation nondeterministic
- We cannot say that \( x = 1 \), we can only say:

\[
\{\text{emp}\} \; x := \text{alloc()} \; \{x = 1 \lor x = 2 \lor \ldots\}
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- After applying the frame rule, this is still true

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\{y \mapsto 2\} \; x := \text{alloc()} \; \{y \mapsto 2 \land (x = 1 \lor x = 2 \lor \ldots)\}
\]

- Problem: OL supports execution models other than nondeterminism
Must and May Properties

- Separation Logic only supports *must* properties
- Outcome Logic also supports *may* properties

\[
\langle \text{ok : emp} \rangle \\
x := \text{malloc}() ; \\
[x] \leftarrow 1 \\
\langle (\text{er : } x = \text{null}) \oplus T \rangle
\]
Even with nondeterministic allocation, the following is valid:

$$\langle \text{ok : emp} \rangle \ x := \text{alloc()} \ \langle (\text{ok : } x = 1) \oplus T \rangle$$

Using the frame rule, we get something that is not true

$$\langle \text{ok : } y \mapsto 2 \rangle \ x := \text{alloc()} \ \langle (\text{ok : } y \mapsto 2 \land x = 1) \oplus T \rangle$$
To make memory allocation *local*, assertions cannot mention heap addresses.

**Lemma:** Outcome Logic assertions are invariant to heap permutations.

\[ \forall \pi. \quad m \models \varphi \implies \pi(m) \models \varphi \]
Safe Preconditions

- Separation Logic requires preconditions to be *safe*
- If not, the following would be valid:
  \[
  \{\text{emp}\} \ [x] \leftarrow 1 \ \{\text{emp}\}
  \]
- Now, using the frame rule, we get
  \[
  \{x \mapsto 2\} \ [x] \leftarrow 1 \ \{x \mapsto 2\}
  \]
- Which is clearly false!
Safety in Outcome Logic

- Safe preconditions are *undesirable* in Outcome Logic
- Bugs do not require us to look at all paths

\[
\langle \text{ok : } x = \text{null} \rangle \ (\ [x] \leftarrow 1) + C \langle \text{er : } x = \text{null} \rangle \oplus T
\]

- If the pre had to be safe, we would need to inspect \( C \)
Outcome Logic + Concurrency
Concurrent Outcomes

- Outcomes correspond to possible interleavings

\[
\begin{align*}
&\langle \text{ok : } x \mapsto \bot \rangle \\
&[x] \leftarrow 1 \parallel [x] \leftarrow 2 \\
&\langle (\text{ok : } x \mapsto 1) \oplus (\text{ok : } x \mapsto 2) \rangle
\end{align*}
\]
The Exchange Law

- From Concurrent Kleene Algebra:

\[(C_1 \parallel C'_1) \bowtie (C_2 \parallel C'_2) \leq (C_1 \bowtie C_2) \parallel (C'_1 \bowtie C'_2)\]

- In Outcome Logic:

\[
\frac{\langle \varphi \rangle (C_1 \parallel C'_1) \bowtie (C_2 \parallel C'_2) \langle \psi \rangle}{\langle \varphi \rangle (C_1 \bowtie C_2) \parallel (C'_1 \bowtie C'_2) \langle \psi \oplus T \rangle}
\]

\text{EXCHANGE}
Concurrent Bugs

The following program will crash in some interleavings

\[
\begin{align*}
[x] & \leftarrow 1 ; \parallel \text{free}(x) ; \\
[x] & \leftarrow 2 \parallel \text{skip}
\end{align*}
\]

The exchange law can partially sequentialize the program

\[\begin{align*}
\text{Exchange} & : \\
& \frac{
\langle \text{ok} : x \mapsto - \rangle ([x] \leftarrow 1 \parallel \text{free}(x)) ; ([x] \leftarrow 2 \parallel \text{skip}) \langle \text{er} : x \not\leftrightarrow \rangle 
}{
\langle \text{ok} : x \mapsto - \rangle ([x] \leftarrow 1 ; [x] \leftarrow 2) \parallel (\text{free}(x) ; \text{skip}) \langle (\text{er} : x \not\leftrightarrow) \oplus T \rangle}
\end{align*}\]

Complete the proof using all the Iris machinery
Conclusion

- OL is sound for correctness and incorrectness with effects
- The OL frame rule allows effects and dropping paths
- Future work: OL + concurrency (with Iris)

- Further reading at cs.cornell.edu/~noamz
  - Outcome Logic: A Unified Foundation for Correctness and Incorrectness Reasoning [OOPSLA’23]
  - Outcome Separation Logic: Local Reasoning for Correctness and Incorrectness with Computational Effects [arXiv]

- A lot more cool stuff!
  - Algebraic semantics based on semirings
  - Relative completeness proof
  - OL subsumes Hoare Logic, probabilistic Hoare Logic, etc
  - Symbolic execution using bi- and tri-abduction