First Steps Towards Probabilistic Iris: Harmonizing Independence, Conditioning, and Dynamic Heap Allocation

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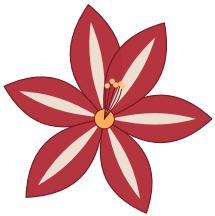

Probabilistic Program Logics

Distributional Probabilistic Logics (DPLs)	**Specialized Probabilistic Logics (SPLs)**
Native **assertions over distribution** of program output/state	Traditional assertions over output/state plus **probabilistic resources**
PSL [Barthe et al. 2020] Lilac [Li et al. 2023] Bluebell [Bao et al. 2025] pcOL [Zilberstein et al. 2026]	Eris [Aguirre et al. 2024] Tachis [Haselwarter et al. 2024] Clutch [Gregersen et al. 2024] Coneris [Li et al. 2025]
$\{\mathit{True}\}\ \mathit{flip}()\ \{V.\ V \sim \mathit{Ber}_{1/2}\}$	$\{\frac{1}{2}(0.5)\}\ \mathit{flip}()\ \{v.v = 1\}$

Probabilistic Program Logics

Distributional Probabilistic Logics (DPLs)	**Specialized Probabilistic Logics (SPLs)**
Native **assertions over distribution** of program output/state	Traditional assertions over output/state plus **probabilistic resources**
Can support **wide range of properties** including (conditional) independence	focus on **specific property** of interest e.g. expected values, error bounds
	often superior language feature support + fancy Iris reasoning techniques + strong modularity

DPLs have potential for being a **general-purpose foundation** for probabilistic logics, but so far are **lagging behind** SPLs in e.g. supported language features

Independence as Separation [Barthe et al. 2020]

Example: one time pad encryption: M message, K key:

$$M \sim Ber_p * K \sim Ber_{1/2}$$

means that M and K are **probabilistically independent**

Separating Conjunction

$\phi * \psi :=$ can separate probability space into independent parts satisfying ϕ and ψ

One time pad encryption correctness:

$$M \sim Ber_p * K \sim Ber_{1/2} \vdash M \sim Ber_p * (K \text{ xor } M) \sim Ber_{1/2}$$

The Conditioning Modality [Li et al. 2023]

Conditioning (discrete setting): probabilistic *case distinction* $C_\mu v.\ \Phi(v)$

$$M \sim Ber_p * K \sim Ber_{1/2} \vdash C_{Ber_p} v. \begin{cases} [M=0] * [K \text{ xor } M = K] * K \sim Ber_{1/2} & \text{if } v = 0 \\ [M=1] * [K \text{ xor } M = \neg K] * K \sim Ber_{1/2} & \text{else} \end{cases}$$

$$\vdash C_{Ber_p} v. \begin{cases} [M=0] * (K \text{ xor } M) \sim Ber_{1/2} & \text{if } v = 0 \\ [M=1] * (K \text{ xor } M) \sim Ber_{1/2} & \text{else} \end{cases}$$

$$\vdash (K \text{ xor } M) \sim Ber_{1/2} * C_{Ber_p} v. \begin{cases} [M=0] & \text{if } v = 0 \\ [M=1] & \text{else} \end{cases}$$

$$\vdash (K \text{ xor } M) \sim Ber_{1/2} * M \sim Ber_p$$

The Gap

All existing logics with support for independence reasoning
do not support dynamic memory allocation
...let alone more sophisticated Iris-style ghost state

The Challenge of Dynamic Allocation

Probabilistic Logics
Separation = independence
$M \sim Ber_p * K \sim Ber_{1/2}$

$+?$

Classic Separation Logic
Separation = heap disjointness
$\ell_1 \mapsto v_1 * \ell_2 \mapsto v_2$

Approach by PSL and pcOL: Separation means independence **and** heap disjointness:

Separating Conjunction in PSL/pcOL

$\phi * \psi \triangleq$ can separate probability space into two independent parts **over disjoint parts of the storage** that satisfy ϕ and ψ

$\Rightarrow$ in PSL/pcOL: $\phi * \psi$ requires heap disjointness **across all random outcomes**

The Challenge of Dynamic Allocation

in PSL/pcOL: $\phi * \psi$ requires heap disjointness across all random outcomes

Definition does not work with dynamic allocation!

Example

$\{\top\}$ (**if** flip() **then ref** 0); (**ref** flip(), **ref** flip()) $\{(L_1, L_2).\ L_1 \rightsquigarrow \mathit{Ber}_{1/2} * L_2 \rightsquigarrow \mathit{Ber}_{1/2}\}$

Assume det. allocator that allocates $\ell_0, \ell_1, \ell_2, \ldots$ in order

$\Rightarrow$ First flip false $\rightarrow (\ell_0, \ell_1)$ returned, first flip true $\rightarrow (\ell_1, \ell_2)$ returned

$\Rightarrow$ ℓ_1 can belong to either L_1 or L_2: no global disjointness!

$\Rightarrow$ Triple is not valid with the PSL/pcOL model of separation!

Key Contributions

Amaryllis

First logic to support independence + **dynamic heap allocation**

- General construction for lifting ownership of Iris resource to probabilistic **ownership per random outcome**
- Instance: Novel model that requires heap disjointness only per random outcome instead over all of them
- Fully mechanized in Rocq, including Iris proof mode

Limitations: Only finite distributions → only terminating programs.
No step indexing, invariants, concurrency

Amaryllis' Model of Per-Outcome Disjointness

Key Idea

Require heap disjointness only **per random outcome** instead over all of them

Amaryllis' Model explicitly tracks random choices (indexed valuations)

RID: set of *random choice identifiers*

Model

$$\mathbb{P}(\mathsf{RID} \to \mathbb{B}) \times RV_{\mathsf{Heap}} \qquad \text{where } RV_T \triangleq (\mathsf{RID} \to \mathbb{B}) \to T$$

$(\phi * \psi)(\mathcal{P}, R) \triangleq$ can separate $\mathcal{P}$ into independent $\mathcal{P}_1, \mathcal{P}_2$ and R into two pointwise disjoint R_1, R_2 s.t. $\phi(\mathcal{P}_1, R_1)$ and $\psi(\mathcal{P}_2, R_2)$

Logic Assertions

Model

$$\mathbb{P}(\mathsf{RID} \to \mathbb{B}) \times RV_{\mathsf{Heap}} \qquad \text{where } RV_T \triangleq (\mathsf{RID} \to \mathbb{B}) \to T$$

$$(V \sim \mu)(\mathcal{P}, R) \triangleq V \text{ is distributed as } \mu \text{ in } \mathcal{P}$$

$$(L \mapsto V)(\mathcal{P}, R) \triangleq \forall \rho \in \mathsf{supp}(\mathcal{P}).\ (L(\rho) \mapsto V(\rho))(R(\rho))$$

$$L \rightsquigarrow \mu \triangleq \exists V.\ V \sim \mu \wedge L \mapsto V$$

Amaryllis Logic

Idea: Use random expression variable in the middle of triple!

	Classic Separation Logic	**Amaryllis**
Points-tos	$\ell \mapsto v$ with $\ell : \mathit{Loc}, v : \mathit{Val}$	$L \mapsto V$ with $L : \mathit{RV}_{\mathit{Loc}}, V : \mathit{RV}_{\mathit{Val}}$
Allocation Rule	$\{\top\}\ \mathbf{ref}\ v\ \{\ell.\ \ell \mapsto v\}$	$\{\top\}\ \rho.\ \mathbf{ref}\ V(\rho)\ \{L.\ L \mapsto V\}$
Load	$\{\ell \mapsto v\}\ !\ell\ \{w.\ v = w\}$	$\{L \mapsto V\}\ \rho.\ !L(\rho)\ \{W.\ V \approx W\}$
Store	$\{\ell \mapsto v\}\ \ell \leftarrow w\ \{\ell \mapsto w\}$	$\{L \mapsto V\}\ \rho.\ L(\rho) \leftarrow W(\rho)\ \{L \mapsto W\}$
Sample	-	$\{\top\}\ \mathrm{flip}(p)\ \{V.\ V \sim \mathit{Ber}_p\}$

Example Verification in Amaryllis

Example

$\{\top\}\ (\textbf{if}\ \mathsf{flip}()\ \textbf{then}\ \textbf{ref}\ 0);\ (\textbf{ref}\ \mathsf{flip}(), \textbf{ref}\ \mathsf{flip}())\ \{(L_1, L_2).\ L_1 \rightsquigarrow \mathit{Ber}_{1/2} * L_2 \rightsquigarrow \mathit{Ber}_{1/2}\}$

Proof idea:
Condition on outcome of first flip and reason **reason case-by-case** using C-lift!

$$\text{C-LIFT}\ \frac{\forall v.\ \{\Phi(v)\}\ E\ \{\Psi(v)\}}{\{C_\mu v.\ \Phi(v)\}\ E\ \{C_\mu v.\ \Psi(v)\}}$$

First, show:

$$\{\top\}\ (\mathbf{ref}\ \mathsf{flip}(), \mathbf{ref}\ \mathsf{flip}())\ \{\exists L_1, L_2.\ V \approx (L_1, L_2) * L_1 \rightsquigarrow \mathit{Ber}_{1/2} * L_2 \rightsquigarrow \mathit{Ber}_{1/2}\}$$

Then:

$$\begin{aligned}
\{\top\}\ (\mathbf{if}\ \mathsf{flip}()\ \mathbf{then}\ \mathbf{ref}\ 0);\ \ldots \quad & \{V.\ \exists L_1, L_2.\ V \approx (L_1, L_2) * \ldots\} \dashv \\
\{V_f \sim \mathit{Ber}_{1/2}\}\ (\mathbf{if}\ V_f\ \mathbf{then}\ \mathbf{ref}\ 0);\ \ldots \quad & \{V.\ \exists L_1, L_2.\ V \approx (L_1, L_2) * \ldots\} \dashv \\
\{C_{\mathit{Ber}_{1/2}} v.\ V_f \approx v\}\ (\mathbf{if}\ V_f\ \mathbf{then}\ \mathbf{ref}\ 0);\ \ldots \quad & \{V.\ C_{\mathit{Ber}_{1/2}} v.\ \exists L_1, L_2.\ V \approx (L_1, L_2) * \ldots\} \dashv \\
\{V_f \approx v\}\ (\mathbf{if}\ V_f\ \mathbf{then}\ \mathbf{ref}\ 0);\ \ldots \quad & \{\exists L_1, L_2.\ V \approx (L_1, L_2) * \ldots\} \dashv \\
\{\top\}\ (\mathbf{if}\ v\ \mathbf{then}\ \mathbf{ref}\ 0);\ \ldots \quad & \{\exists L_1, L_2.\ V \approx (L_1, L_2) * \ldots\}
\end{aligned}$$

Do case analysis on v:

$$\begin{aligned}
\{\top\}\ (\mathbf{if}\ \mathsf{true}\ \mathbf{then}\ \mathbf{ref}\ 0);\ \ldots \quad & \{\exists L_1, L_2.\ V \approx (L_1, L_2) * \ldots\} \dashv \\
\{\top\}\ \mathbf{ref}\ 0;\ (\mathbf{ref}\ \mathsf{flip}(), \mathbf{ref}\ \mathsf{flip}()) \quad & \{\exists L_1, L_2.\ V \approx (L_1, L_2) * \ldots\} \dashv \\
\{L \mapsto 0\}\ (\mathbf{ref}\ \mathsf{flip}(), \mathbf{ref}\ \mathsf{flip}()) \quad & \{\exists L_1, L_2.\ V \approx (L_1, L_2) * \ldots\}
\end{aligned}$$

Example Verification in Amaryllis

Example

$\{\top\}\ (\textbf{if}\ \mathsf{flip}()\ \textbf{then}\ \textbf{ref}\ 0);\ (\textbf{ref}\ \mathsf{flip}(), \textbf{ref}\ \mathsf{flip}())\ \{(L_1, L_2).\ L_1 \rightsquigarrow \mathit{Ber}_{1/2} * L_2 \rightsquigarrow \mathit{Ber}_{1/2}\}$

Key Proof Step

In Amaryllis Model, the postcondition is **convex**:

$$C_{\mathit{Ber}_{1/2}} v.\ \exists L_1, L_2.\ V \approx (L_1, L_2) * \cdots \vdash \exists L_1, L_2.\ V \approx (L_1, L_2) * \ldots$$

Convexity in general not trivial, also challenging on paper

To ease this: Amaryllis' proof mode automates away std cases like this via typeclasses

Summary

- Amaryllis combines independence, conditioning, and dynamic allocation in one logic
- Key idea to combine separation=independence and separation=heap disjointness:
  Require only **per-outcome disjointness** of the heap
- In Amaryllis' Hoare triples $\{\Phi\}\ E\ \{V.\ \Psi(V)\}$, E and V are **random variables**
- Amaryllis' model is parametric in resource algebra, not specific to heaps

What else is in the paper?

- Generalize Iris' update modality to DPL setting
  - Novel notion: *probabilistic frame-preserving update* that preserves conditionings
  - General concept for achieving modular reasoning in DPLs
- First to apply Iris methodology to a DPL:
  - Re-interpret *authorative resource algebra* for DPL setting
  - Define *base logic* (including conditioning modality) then derive *weakest precondition modality* and *program logic* on top