Verifying Wait-Freedom for Concurrent Higher-Order Programs

ECOOP'26

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June 8, 2026

Wait-freedom: the strongest progress guarantee

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let new_cnt_blocking () =
  let c = ref 0 in
  let l = new_lock () in
  let i _ =
    acquire l;
    let n = !c in
    c := n + 1;
    release l;
    n
  in { incr = i }
```

✗

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let new_cnt_lockfree () =
  let c = ref 0 in
  let rec i _ =
    let n = !c in
    if CAS(c, n, n + 1)
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```
let new_cnt_waitfree () =
  let c = ref 0 in
  let i _ = FAA(c, 1)
  in { incr = i }
```

✓

Wait-free implementations are the hardest

Blocking queue, Vindum and Birkedal (2021)

```
let cg_enqueue lock head x =
  let rec enqueue_go x head =
    match head with
    | None
-> Some (x, None)
    | Some p -> Some (fst p, enqueue_go x (snd p))
  in
  acquire lock;
  head := enqueue_go x !head;
  release lock

let cg_dequeue lock head () =
  acquire lock;
  let v = match !head with
          | None
-> None
          | Some p -> head := snd p; Some (fst p)
  in
  release lock;
  v

let cg_queue () =
  let head = ref None in
  let lock = newlock () in
  (cg_dequeue lock head, cg_enqueue lock head x)
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```

Lock-free Michael-Scott queue, version by Vindum and Birkedal (2021)

```
let get_value x =
  match x with
  | None
-> let rec spin () = spin () in spin ()
  | Some v -> v

let ms_dequeue toSent toTail =
  let rec loop () =
    let head  = !toSent in
    let tail  = !toTail in
    let toNext = snd (get_value !head) in
    let p     = NewProph in
    let next  = !toNext in
    if head = (Resolve (!toSent) p ()) then
      if head = tail then
        match !next with
        | None   -> None
        | Some _ -> CAS toTail tail next; loop ()
        end
      else
        if CAS toSent head next then
          Some (get_value (fst (get_value !next)))
        else loop ()
    else loop ()
  in loop ()

let ms_enqueue toTail x =
  let node = ref (Some (Some x, ref (ref None))) in
  let rec loop () =
    let tail  = !toTail in
    let toNext = snd (get_value !tail) in
    let next  = !toNext in
    if tail = !toTail then
      match !next with
      | None   ->
        if CAS toNext next node then
          (CAS toTail tail node; ())
        else loop ()
      | Some _ -> CAS toTail tail next; loop ()
      end
    else loop ()
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let ms_queue () =
  let node
= ref (Some (None, ref (ref None))) in
  let toTail = ref node in
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  ((ms_dequeue toSent toTail), (ms_enqueue toTail))
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  let toTail = ref node in
  let toSent = ref node in
  ((ms_dequeue toSent toTail), (ms_enqueue toTail))
```

Wait-free queue, Kogan and Petrank (2011) (translated w/ Claude)

```
type node
= { mutable value: int; mutable next: node option ref;
                mutable enqTid: int; mutable deqTid: int }
type op_desc = { phase: int; pending: bool; enqueue: bool; node: node option }

let head
= ref (mk_sentinel ())
let tail
= ref (mk_sentinel ())
let state = Array.init num_thrds
    (fun _ -> { phase = -1; pending = false; enqueue = true; node = None })

let max_phase () =
  Array.fold_left (fun m d -> max m d.phase) (-1) state

let is_still_pending tid ph =
  state.(tid).pending && state.(tid).phase <= ph

let help phase =
  Array.iteri (fun i d ->
    if d.pending && d.phase <= phase then
      if d.enqueue then help_enq i phase
      else
help_deq i phase
    ) state

let enq value =
  let phase = max_phase () + 1 in
  state.(tid) <- { phase; pending = true; enqueue = true;
                   node = Some { value; next = ref None;
                                 enqTid = tid; deqTid = -1 } };

  help phase;
  help_finish_enq ()

let help_enq tid phase =
  while is_still_pending tid phase do
    let last = !tail in
    let next = !(last.next) in
    if last = !tail then
      match next with
      | None ->
        if is_still_pending tid phase then
          if CAS last.next None (state.(tid).node) then
            help_finish_enq ()
      | Some _ ->
        help_finish_enq ()
  done

let help_finish_enq () =
  let last = !tail in
  match !(last.next) with
  | None -> ()
  | Some next ->
      CAS state.(tid) cur_desc { cur_desc with pending = false };
      CAS tail last next
    end
let deq () =
  let phase = max_phase () + 1 in
  state.(tid) <- { phase; pending = true; enqueue = false; node
  help phase;
  help_finish_deq ();
  match state.(tid).node with
  | None     -> raise Empty
  | Some node -> node.next.value

let help_deq tid phase =
  while is_still_pending tid phase do
    let first = !head in
    let last  = !tail in
    let next
= !(first.next) in
    if first = !head then
      if first = last then
        match next with
        | None ->
          let cur_desc = state.(tid) in
          if last = !tail && is_still_pending tid phase then
            CAS state.(tid) cur_desc
                { cur_desc with pending = false; node = None }
        | Some _ -> help_finish_enq ()
      else
        let cur_desc = state.(tid) in
        let node
= cur_desc.node in
        if is_still_pending tid phase then begin
          if first = !head && node <> Some first then
            if not (CAS state.(tid) cur_desc
                    { cur_desc with node = Some first }) then
          else begin
            CAS first.deqTid (-1) tid;
            help_finish_deq ()
          end
        end
  done

let help_finish_deq () =
  let first = !head in
  match !(first.next) with
  | None -> ()
  | Some next ->
    let tid = first.deqTid in
    if tid <> -1 then begin
      let cur_desc = state.(tid) in
      if first = !head then begin
        CAS state.(tid) cur_desc { cur_desc with pending = fals
```

Wait-freedom is hard to even define

"An operation is wait-free if it guarantees that every call finishes its execution in a finite number of steps." [Herlihy and Shavit (2012)]

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Is the data structure initialized?

What if operation was stored in heap?

How to define formally in an ML-like language?

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What about safety?

Who calls it? Is it an arbitrary client program?

How the execution is scheduled?

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$$waitFree_{\mathfrak{C}}(\mathsf{op}) : Prop$$

need to account for initialization; omit this parameter for simplicity

Wait-freedom is hard to even define, harder to prove.

$\vdash \mathit{waitFree}(\mathsf{op})$

Wait-freedom is hard to even define, harder to prove. Solution: verify the operation in Iris instead

$\vdash \mathsf{WaitFreeSpec}(\mathsf{op})$ $\Rightarrow$ $\vdash \mathit{waitFree}(\mathsf{op})$

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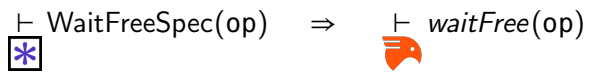

Our contributions, all mechanized in Rocq:

- Definition *waitFree*(op) of op's wait-freedom in higher-order setting;
- Iris-based specification WaitFreeSpec(op) and the corresponding adequacy theorem;
- Case studies for simple algorithms;
- Adapting the approach for verifying realistic, *not exactly wait-free* algorithms

Iris Workshop'25: termination proofs with Lawyer logic

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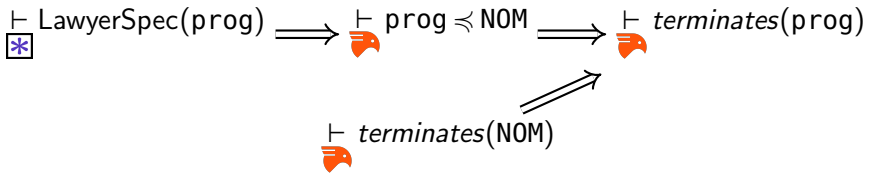
Lawyer approach, restricted to non-blocking concurrency:



Iris Workshop'25: termination proofs with Lawyer logic

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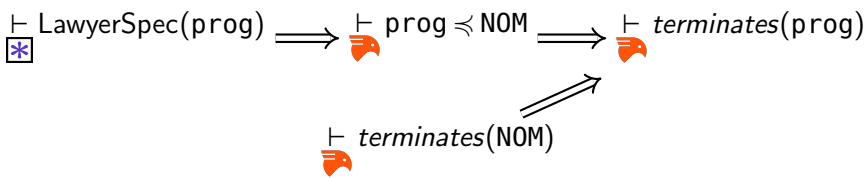

LTS	Language	NOM ("No-Obligations Model")
State	thread pool and heap	finite *fuel* assigned to each thread
Labels	thread ids	thread ids
Transitions	small-step semantics	besides others, *burning fuel on every step*

Not done yet: specs imply weaker property for open programs

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Key property for refining closed `prog` to `NOM`

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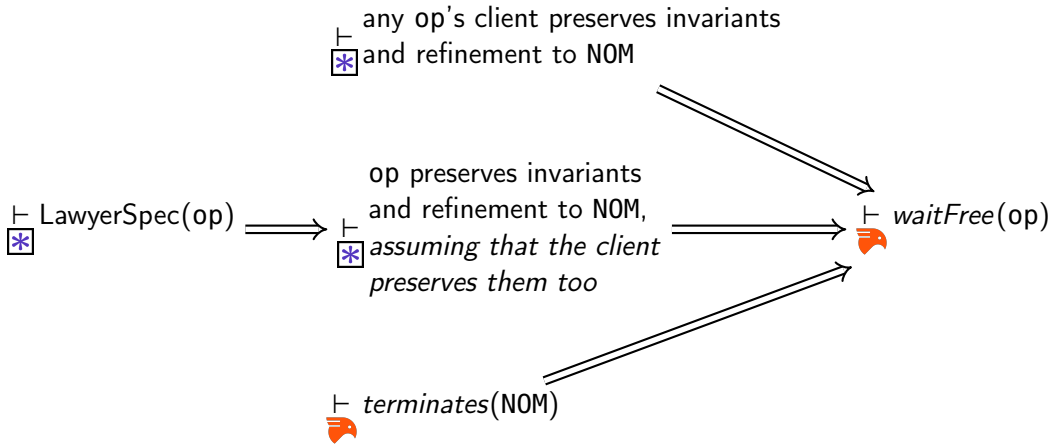
turns into a weaker property for standalone wait-free `op`:

$\vdash_{\boxed{*}}$ LawyerSpec(`op`) $\Longrightarrow$ $\vdash_{\boxed{*}}$ execution of `op` preserves invariants and refinement to `NOM`, *assuming that client of* `op` *preserves them too*

User proves spec and applies our new adequacy theorem



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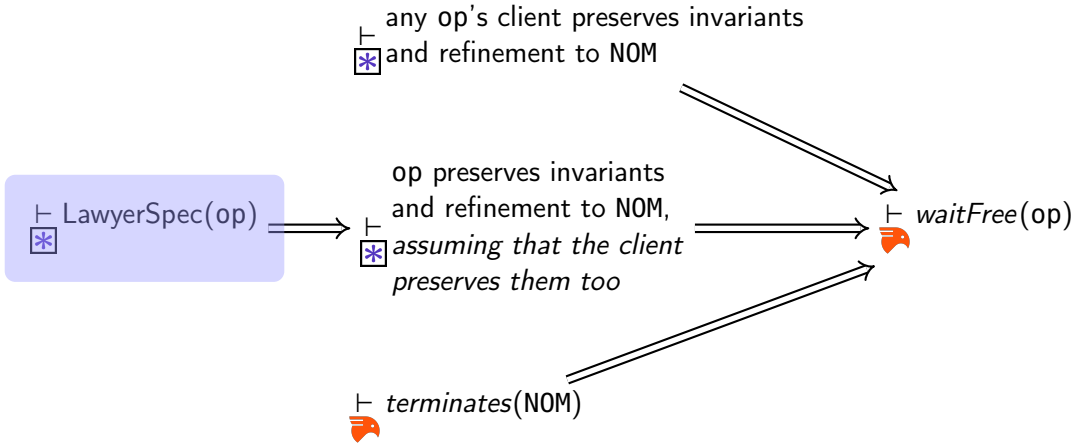

Reusing Lawyer logic to prove wait-freedom specification

```
let incr x =

    FAA(c, 1)
```

Reusing Lawyer logic to prove wait-freedom specification

Key part of proving $\vdash$ LawyerSpec(`incr`): $\boxed{*}$

For every τ : Thread
and x : *Val*:

$\{ \text{DB}_\tau * \text{DB}_\tau * \boxed{\exists n : \mathbb{Z}.\, c \mapsto n}^{\mathcal{N}} \}^{\tau}$

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Amount of fuel and concrete invariant depend on operation

"phase" resource omitted for simplicity

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$\{\exists_\tau\}$

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{True}

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$$\textsc{Lawyer-beta-fuel}\ \frac{\{P\}\ e[v/x]\ \{Q\}^\tau_{\mathcal{E}}}{\{\exists_\tau * P\}\ (\lambda x.\ e)\, v\ \{Q\}^\tau_{\mathcal{E}}}$$

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    $\{\}_{\tau}^{\top \setminus \mathcal{N}}$
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    $\{\exists n.\, c \mapsto n\}^{\top \setminus \mathcal{N}}$
  {True}
{True}

LAWYER-BETA-FUEL

$$\frac{\{P\}\ e[v/x]\ \{Q\}^{\tau}_{\mathcal{E}}}{\{\}_{\tau} * P\}\ (\lambda x.\ e)\, v\ \{Q\}^{\tau}_{\mathcal{E}}}$$

IRIS-INV-ACC-TL

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LAWYER-FAA-FUEL

$$\{\exists_\tau * \ell \mapsto n\}\ \mathsf{FAA}(\ell, k)\ \{m.\, m = n * \ell \mapsto (n+k)\}^{\tau}_{\mathcal{E}}$$

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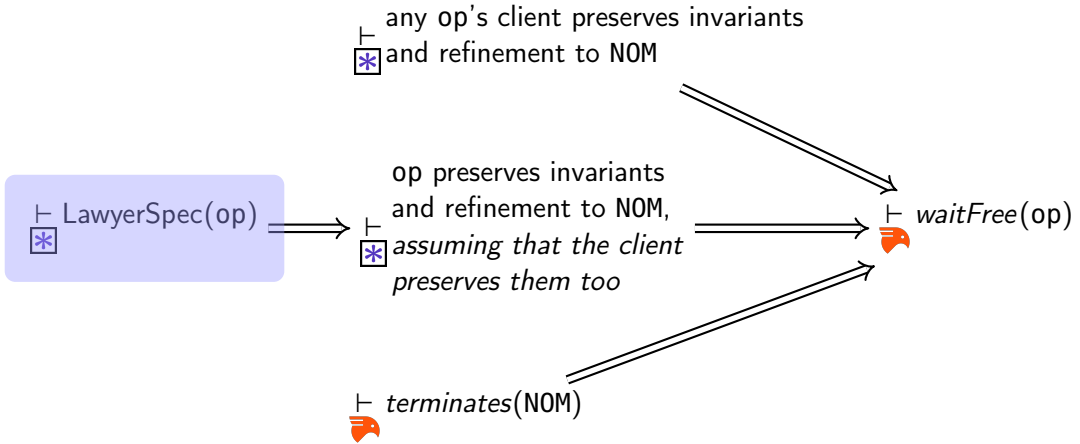

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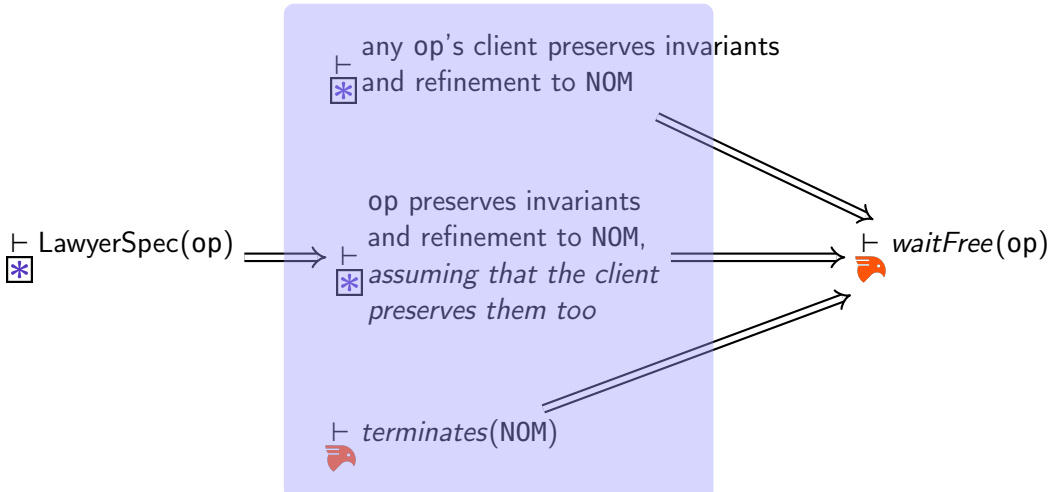

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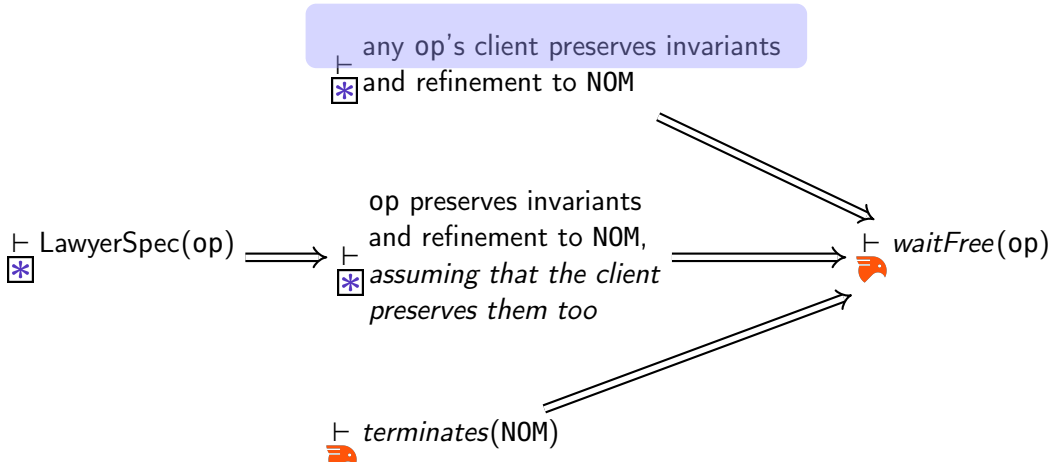

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E.g. safe termination of `let incr x = FAA(c, 1)`
relies on $\boxed{\exists n : \mathbb{Z}.\, c \mapsto n}$ being preserved by its client

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op's client

Robust safety of verified `op`, more formally:

$\vdash \forall \texttt{prog} : \mathit{Expr}.\ \mathsf{LawyerSpec}(\texttt{op}) \mathrel{-\!\!*} \mathsf{PresInv}(\texttt{prog}[\texttt{op}/x])$

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How to prove it for any `prog`?

In fact, most of clients preserve operation's invariants

What preserves the invariant $\boxed{\exists n : \mathbb{Z}.\, c \mapsto n}$ of `incr`?

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incr ()
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                  ||  let n = 5 + 5 in  ||
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                ||  let n = 5 + 5 in  ||  while true do
incr ()         ||  incr n;           ||    incr ()
                ||                    ||  done
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- Writing to *arbitrary* locations ✗

Prove robust safety with logical relations

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$\mathrm{RS_E}(\texttt{prog} : \mathit{Expr}) \approx$

execution of `prog` preserves invariants,
might get stuck,
and the returned value (if any) is "safe"

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$\boxed{*}$

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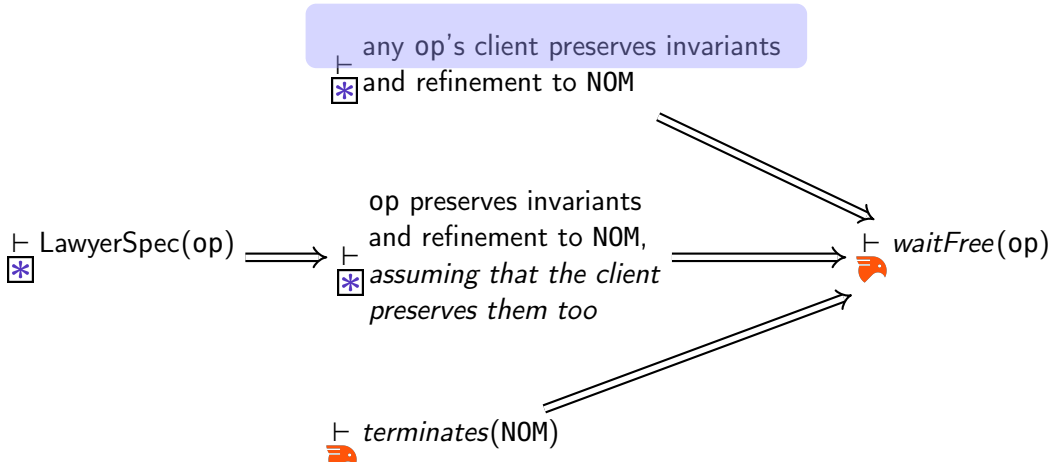
and conclude robust safety of verified `op`:

$$\vdash \boxed{*}\ \forall \texttt{prog}.\ \mathit{NoPtrArith}(\texttt{prog}) \mathrel{-\!\!*} \mathrm{RS_V}(\texttt{op}) \mathrel{-\!\!*} \mathrm{RS_E}(\texttt{prog}[\texttt{op}/x])$$

by adapting the proof of LawyerSpec(`op`)

User proves spec and applies our new adequacy theorem



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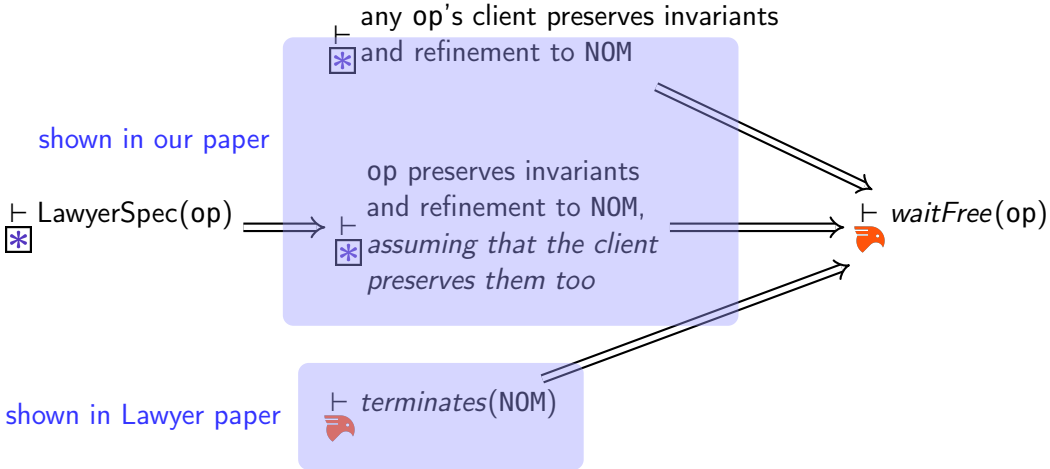

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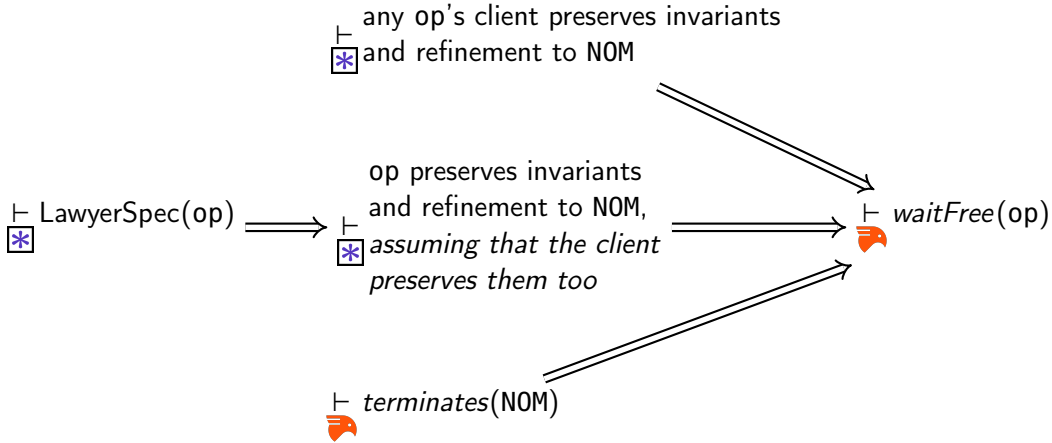

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- Realistic wait-free data structures restrict concurrency ...

Realistic wait-free data structures restrict concurrency

```
void enq(int value) {
  long phase = maxPhase() + 1;
  state.set(TID, new
    OpDesc(phase, true, true, new Node(value, TID)));
  help(phase);
  help_finish_enq();
}
```

1. Post its own operation
2. Help previously posted operations
3. Complete its own operation

```
AtomicReferenceArray<OpDesc> state;

WFQueue () {
  state = new AtomicReferenceArray<OpDesc>(NUM_THRDS);
  ...
```

Adapting our approach to restricted wait-freedom

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Result for 1-enqueuer, 1-dequeuer queue by Jayanti and Petrovic (2005):

$\vdash$ *waitFreeRestr*([enqueuer, dequeuer])

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$$\mathit{waitFreeRestr}(\mathsf{ops} : \mathit{List}\,\mathit{Val}) \approx \begin{array}{l}\text{similar to } \mathit{waitFree}(\mathsf{ops}[i]) \text{ for every } i,\\ \text{but } i\text{th thread can only call } \mathsf{ops}[i];\\ \text{forks are prohibited, but } \mathsf{ops}[i] = \mathsf{ops}[j] \text{ is allowed}\end{array}$$

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waitFreeRestr(ops : *List Val*) $\approx$ similar to *waitFree*(ops[i]) for every i, but ith thread can only call ops[i]; forks are prohibited, but ops[i] = ops[j] is allowed

WaitFreeRestrSpec(ops) $\approx$ for each op $\in$ ops, show $\forall a, \tau.\ \{\Diamond_\tau * \ldots * \Diamond_\tau * \mathrm{tok}_{\mathrm{op}}\}\ \mathrm{op}\ a\ \{\mathrm{tok}_{\mathrm{op}}\}^\tau$ assuming there are only *count*(op, ops) many $\mathrm{tok}_{\mathrm{op}}$

$\vdash$ WaitFreeRestrSpec(ops) $\Rightarrow$ $\vdash$ *waitFreeRestr*(ops)

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$$\mathsf{WaitFreeRestrSpec}(\mathsf{ops}) \approx \text{for each } \mathsf{op} \in \mathsf{ops}, \text{ show } \forall a, \tau.\ \{\boxminus_\tau * \ldots * \boxminus_\tau * \mathsf{tok}_{\mathsf{op}}\}\ \mathsf{op}\, a\ \{\mathsf{tok}_{\mathsf{op}}\}^\tau$$

assuming there are only $\mathit{count}(\mathsf{op}, \mathsf{ops})$ many $\mathsf{tok}_{\mathsf{op}}$

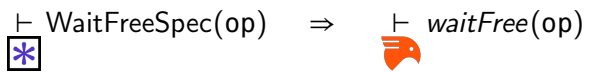
$\vdash \mathsf{WaitFreeRestrSpec}(\mathsf{ops}) \quad \Rightarrow \quad \vdash \mathit{waitFreeRestr}(\mathsf{ops})$

Logical relation and adequacy proof change, but the program logic stays the same!

Future work

- Linearizability in the same framework (requires prophecy variables in Trillium)
- Verification of universal constructions for wait-freedom
- Minor changes, *e.g.*, being more flexible with amount of fuel

Wait-freedom is hard to even define, harder to prove. Solution: verify the operation in Iris instead



Our contributions, all mechanized in Rocq:

- Definition *waitFree*(op) of op's wait-freedom in higher-order setting;
- Iris-based specification WaitFreeSpec(op) and the corresponding adequacy theorem;
- Case studies for simple algorithms;
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Thank you!

Backup slides

Logical relation

$$\mathsf{RS_E}(e) \triangleq \forall \tau.\ \{\mathsf{True}\}^{\lightning}\ e\ \{v.\mathsf{RS_V}(v)\}^{\tau}$$

$$\mathsf{RS_V}(()) \triangleq \mathsf{RS_V}(b : \mathbb{B}) \triangleq \ldots \triangleq \mathsf{True}$$

$$\ldots$$

$$\mathsf{RS_V}((v_1, v_2)) \triangleq \mathsf{RS_V}(v_1) * \mathsf{RS_V}(v_2)$$

$$\mathsf{RS_V}(\ell : \mathit{Loc}) \triangleq \boxed{(\exists v.\ \ell \mapsto v * \mathsf{RS_V}(v)) \vee \mathsf{freed}(\ell)}$$

$$\mathsf{RS_V}(\lambda\, x.e) \triangleq \forall \tau, v.\ \{\mathsf{RS_V}(v)\}^{\lightning}\ (\lambda\, x.e)v\ \{r.\mathsf{RS_V}(r)\}^{\tau}$$

The $\lightning$ triples are different from those used in Lawyer proofs:

- they don't enforce model refinement, *i.e.* only consider safety
- they allow to get stuck

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